elementary in the sense that no excessive demands are made on the reader's knowledge at the outset. Here is introduced the canonical form of equations which is destined to play a predominant part in the sequel, and earlier is a sketch of the theory of linear differential equations with periodic coefficients as developed by Hermite, Floquet and Poincaré.

In the second section, the partial differential equation of Jacobi is discussed and Stäckel's important theorem on the possibility of solving it by separating the variables is given. This leads to the consideration of motions determined by one degree of freedom, and in particular of forms of motion termed "Libration" and "Limitation," the latter being of the nature of Poincaré's asymptotic solutions. An account follows of conditionally periodic motions, based on the researches of Staude.

The third section treats of the motion of a particle attracted by two fixed Newtonian centres of force. This problem provides illustrations of the theory of the previous section. Otherwise it is a little inconsistent with the practical aim of the author, for its astronomical interest, as was frankly admitted by Lagrange, is very slight. A reference might here have been made to Prof. Greenhill's paper on the stability of such forms of motion (*Proc.* Lond. Math. Soc., vol. xxii.).

The problem of two bodies is treated in some detail in the fourth section, the Hamilton-Jacobi equation being made the basis of the discussion. The case of a repulsive force is also discussed, and this leads to a digression on the dynamical theory of the tails of comets.

The most important results in the general problem of three bodies are investigated in the fifth section. The general integrals and the different forms which they assume when expressed in different systems of coordinates are discussed. The method of variation of parameters is explained in conjunction with Jacobi's canonical elements and also in connection with relative coordinates. The chief results of Jacobi's classical memoir on the elimination of the nodes and of Laplace's theory of stability are given here. Finally, the equations of the problem are reduced to the form expressing four degrees of freedom.

The rest of the book is devoted to the theory of perturbations. In the sixth section, Poincaré's system of canonical elements is introduced, the form of the development of the disturbing function is described and a very brief sketch of Laplace's coefficients is given. The final section contains the theory of secular perturbations of a planetary system, which is treated in some detail. At the end of the volume will be found some useful numerical tables.

The second volume is promised for next year. It will contain the theory of periodic orbits in the problem of three bodies and researches on the convergence of series.      H. C. P.

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## OUR BOOK SHELF.

*Lexikon der Kohlenstoff-Verbindungen.* By M. M. Richter. Pp. 2482. (Hamburg : Leopold Voss, 1899.) Price, 39 parts, 1.80 marks each.

NOTHING could illustrate more forcibly the rapid growth of organic chemistry than the increased dimensions of the new edition of Richter's " Tabellen der Kohlenstoff-

Verbindungen," published in 1884, which now appears for the first time under the title of " Lexicon."

The first edition, the publisher tells us, accounted for 16,000 compounds ; in the present volume, which is brought down to the first quarter of 1899, 67,000 compounds are described, so that in fifteen years organic chemistry may be said to have multiplied more than fourfold. It is not surprising to learn that the stupendous labour of collecting and arranging this enormous mass of material has taken ten years to complete.

The lexicon contains all the known carbon compounds, arranged in order of their molecular formulæ on an ingenious system, which is fully set forth in the introduction. The name and a few physical constants are given, but the chief information is contained in the very full references to the original literature and to Beilstein's well-known " Handbuch." Since the first edition of the book appeared, the nomenclature of the Geneva Commission has been introduced, and in many cases the new and the old names appear side by side.

There is also an index of the names of the different compounds at the end of the volume.

Where organic research is being pursued with the almost feverish rapidity which is in vogue, more especially in the German laboratories, involving in the process the production of many compounds, both old and new, it is easy to understand the time and trouble which might be expended in fixing the identity of these compounds. One object of the lexicon is to lighten the labours of the investigator in this direction.

This becomes more imperative where the number of isomerides is large, for it is not uncommon to meet with 50, 60, or even 100 substances with the same molecular formula. For example, an experimenter who happened in the course of his research to obtain a compound of the formula $C_7H_{10}O_4$ would be confronted with a choice of 59 substances among compounds already known. By reference to the lexicon, he would see from the physical properties whether the compound had already been prepared, or, failing this, he could at once refer to the literature on the subject.

Richter's " Tabellen " is sufficiently well known among chemists and its utility long enough proved to ensure an excellent reception for the new edition and to render superfluous any further description of its use or its merits.

The author complains (and who does not) of the present system, or lack of system, of chemical nomenclature.

Organic chemistry has, in fact, outgrown its mother-tongue. It can no longer express itself clearly in the language of its childhood. An attempt was made by the Geneva Commission of 1892 to introduce reforms, and some excellent proposals were made, and have since been to some extent adopted on the continent. The author adds, "it is and remains deplorable, the fact that the resolutions arrived at at Geneva have no prospect of being generally adopted." It is to be hoped that before many more thousands have been added to the still growing number of organic compounds, the confusion which is rapidly impending through the want of a universally recognised system of nomenclature will be averted by a complete and thorough revision, more especially of the names of ring compounds.

     J. B. C.

*Ueber Harmonie und Complication.* By Dr. Victor Goldschmidt. Pp. 136 ; with 28 figures. (Berlin : Julius Springer, 1901.) Price 4 marks.

MANY attempts have been made to associate the forms occurring in music with forms which manifest themselves to senses other than that of hearing. If the term "harmony" is used to include all such groupings and arrangements as give us pleasure, then we have harmonies

of form, harmonies of colour, and so forth. Dr. Gold-schmidt's object appears to us to be to reduce all such harmonies to a common formula, and he considers that the different kinds of harmony are governed by a common law, the " law of complication."

As an example of the arguments employed, a series of numbers is obtained from the intervals of the musical scale which coincide with numbers in another series alleged to be obtainable from crystallography. But the numbers in the case of the music do not represent actual intervals, but are derived from them by a homographic transformation, according to which the keynote and its octave are represented by o and ∞ and the major fifth by 1, and the series is incomplete unless the minor seventh be included in the list. And the identity of the two series is by no means complete ; for there are terms in the series derived from music which are absent from that obtained from crystallography.

It is easy to find connections as close as those dealt with in the present work between phenomena which have nothing whatever in common. For a considerable period, the number of wranglers in the Cambridge mathematical tripos was observed to be intimately related to the fre-quency of sun-spots, and anyone who should seek to establish a connection between the notes of the musical scale and the courses of a *table d'hôte* dinner might easily make out a very strong case. What is most sur-prising is that the analogy which *a priori* exists between musical intervals and colour intervals, both of which depend on ratios of vibration-frequency, appeals but little to our senses, so little, in fact, that certain writers have even sought to establish relations between chords and colours quite independently of the known relations of pitch. As for the connection which no doubt exists between a love of music and a talent for mathematics, its cause is not difficult to find. A mind like that of Beltrami, who could discover in the purely abstract ideas of geometry and algebra truths applicable to spaces other than that in which we live, was necessarily well trained to appreciate that beauty of form dissociated from worldly matters which exists in the sonatas and sym-phonies of the older composers. In order, on the other hand, to make it more palatable to a mind that wants to grasp something tangible, music is commonly associated with such mundane ideas as love, vice, battle and murder, and sudden death, the triumph of the victorious, the wails of the vanquished.

*Opere Matematiche di Francesco Brioschi.* Vol. ii. *Opere Matematiche di Eugenio Beltrami.* Vol. i. Pp. 456 and 437. (Milan : Ulrico Hoepli, 1902.) Price 25 lire.

THE second volume of Brioschi's works contains thirty-five papers contributed to the *Annali di Matematica pura ed applicata*, series 1 and series 2, vols. i.–xiv., between the years 1858 and 1887. These papers have all been carefully revised by Profs. Cerruti (Rome), Gerbaldi (Palermo), Loria (Genoa), Pascal (Pavia), Pittarelli (Rome), Reina (Rome) and Tonelli (Rome). A considerable number of them deal with linear dif-ferential equations, but elliptic and hyperelliptic functions, curvilinear coordinates, binary forms and many other subjects are treated ; and the papers also include obituary notices of Borehardt and Chasles.

After the death of Prof. Beltrami, in 1900, the Faculty of Science of the University of Rome resolved to estab-lish a memorial of the distinguished mathematician, and it was decided that the most fitting form for the memorial would be a complete edition of Beltrami's col-lected works ; to quote Prof. Tonelli, *monumentum aere perennius.* In this case, the work of preparing the volumes has been carried out entirely under the direc-tion of Profs. Cremona, Castelnuovo and Tonelli, as representatives of the Roman Faculty of Science, who have been aided by the collaboration of Profs.

Bianchi, Burgatti, Cerruti, Dini, Pittarelli, Reina and Volterra. The order of arrangement differs from that adopted for Brioschi's works. Instead of being grouped according to journals, Beltrami's papers are arranged in strict chronological order, and this volume represents the work of eight years, from 1861 to 1868. That these first eight years of Beltrami's career as a mathematician were productive of work of great value is shown by the list of titles, which include researches on analysis applied to geometry, the flexure of ruled surfaces, resolution of the problem of transforming geodesics on a surface into straight lines in a plane, complex variables on any sur-face, fundamental theories of space of constant curva-ture, and last, but not least, the " Saggio d'interpretazione " of non-Euclidean geometry. The portrait of Beltrami which forms the frontispiece is due to Prof. Pittarelli.

Beltrami's works are published in uniform style with those of Brioschi, and both are printed by the Mathe-matical Press, of Palermo.

*Handbook of the Trees of New England.* By L. L. Dame and Henry Brooks. Pp. xv + 196. (Boston, U.S.A. : Ginn and Co., 1902.)

THE interest connected with the flora of the New England States lies in the fact that situated between Canada and the Alleghany Mountains they furnish the meeting point of a northern and a more southern flora. Since the book is limited to such a relatively small part of the country, it does not possess the general interest which would attach to one which included, for instance, the trees of all the eastern States. What it loses in comparative value, perhaps it gains in definiteness ; it contains useful and succinct descriptions, good illustra-tions specially drawn, and states the horticultural value of all the indigenous species. The Latin nomenclature is satisfactory and correct, except in the case of a species of Acer, and for *Quercus Muhlenbergii,* which is considered by some authorities to be a variety of *Quercus prinus* ; but the popular names are in utter confusion, and we cannot agree with the authors that it is wiser "to record what is, and not what ought to be." Taking *Populus balsamifera* as an illustration, the names recorded are "Balsam. Poplar. Balm of Gilead." Now this tree is certainly not a balsam, and *Populus candicans* is the real Balm of Gilead ; while the name balsam-poplar would be sensible and correct. Apart from this and within its limits, the book may be recom-mended either to enable one to identify the trees or to ascertain their characteristics. English readers will find that only about half-a-dozen species are the same as those indigenous to this country.

*Lake-Country Rambles.* By William T. Palmer. Pp viii + 334. (London : Chatto and Windus, 1902.) Price 6s.

MR. PALMER has here collected a series of papers he has from time to time contributed to various magazines. For many years the author has been a rambler in the lake-country, and has learned to love its inhabitants and to study its varied scenes. The essays are good ex-amples of descriptive writing, but the aspects of nature and the incidents of outdoor life are treated rather from the point of view of the general observer than that of the inquiring naturalist.

*Junior Arithmetic Examination Papers.* Arranged by W. S. Beard. Pp. vi + 106. (London : Methuen and Co., 1902.) Price 1s.

THE ninety examination papers contained in this col-lection cover all the parts of arithmetic generally studied in schools. The first third of the papers gradually in-crease in difficulty from paper 1, on the first four rules, to paper 30, on the mensuration of rectangular solids. The remaining papers are made up of mixed questions and are all well graduated. The questions should be useful to teachers.